Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

the 15. Thus every triad has an odd number, 3 or 1, of the *odd set* of 15, and an even number, 0 or 2, of the *even set* of 16. Any one of the odd set is found therefore in triads with 8 pairs from the even set, and these pairs can be arranged in 15 columns, an 8 by 15 array. Every odd element is found also with 7 pairs from the odd set. This leads to the tabulation of 15 columns of 7 pairs each, ranged above the columns of the 8 by 15 array. Every column is marked then by one odd element above it; the upper partial columns exhibit the head, or Δ_{15}.

Head and array form a convenient mode for constructing Δ_{31}'s that are to have odd-and-even structure. If the head, the Δ_{15}, is itself headless, this tabulation is unique for that Δ_{31}. *I study here exclusively these odd-and-even* Δ_{31}*'s whose head is a headless* Δ_{15}. Given any one such Δ_{31}, tabulated, many others can be obtained by shifting the columns of its 8 x 15 array while the head is kept stationary. To apply this method and to count the distinct Δ_{31}'s that will be produced, one must know the groups $G_{d'}$ and $G_{d'}$, belonging to the head and to array respectively. *The number of resulting* Δ_{31}*'s is certainly not less than 15! divided by the product, d d', of the orders of the groups belonging to the head and to the array respectively.* These orders are small, whence the resulting Δ_{31}'s are very many.

Incidentally, if d and d' are relative primes, the resulting Δ_{31}'s must be of the peculiar kind having no automorphic substitutions; i.e., their group is reduced to the identity. Such cases occur, e.g., with $d = 2$ and $d' = 3$. Full details are to appear in the *Transactions of the American Mathematical Society* for January, 1915.

THE Φ-SUBGROUP OF A GROUP OF FINITE ORDER

By G. A. Miller

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS

Presented to the Academy, November 19, 1914

A set of λ operators $s_1, s_2, .., s_\lambda$ of a finite group G is called a set of generators of G provided there is no subgroup in G which includes each of these operators. When these operators satisfy the additional condition that G can be generated by no $\lambda - 1$ of them the set is said to be a *set of independent generators of G.* Those operators of G which can appear in none of its possible sets of independent generators constitute a characteristic subgroup, which was called by G. Frattini *the* ϕ*-subgroup of G.* See *Rend. Acc. Lincei,* ser. 4, 1, 281 (1885).

Among the results of the present paper which are supposed to be new are the following: The number of the different operators in each of the possible sets of independent generators of a group whose order is a power of a prime number is the same,—that is, if the order of a group is a power of a prime number, the number of its independent generators is an invariant of the group. The ϕ-subgroup of every direct product is the direct product of the ϕ-subgroups of the factors of this direct product. In every group whose order is a power of a prime number the ϕ-subgroup includes the commutator subgroup of the group. By constructing the ϕ-subgroup of a group and of its successive ϕ-subgroups we can always arrive at the identity.

The ϕ-subgroup of a Sylow subgroup of the symmetric group of degree n is the commutator subgroup of this Sylow subgroup. In particular, the number of the different operators in a set of independent generators of the Sylow subgroup of order p^m of the symmetric group of degree p^n, p being any prime number, is n. In particular, the subgroup of order p^{p+1} which is contained in the symmetric group of degree p^2 has exactly 2 independent generators irrespective of the prime number represented by p. No ϕ-subgroup can contain a complete set of conjugate subgroups, or a complete set of conjugate operators, involving more than one subgroup or more than one operator, when this complete set of conjugates is also a complete set of conjugates under the entire group. An important special case of this theorem is that every ϕ-subgroup whose order is not a power of a prime number is the direct product of its Sylow subgroups. This special case was noted by G. Frattini in the article to which reference has been given.

With respect to abelian groups a set of λ independent generators is commonly defined so that the group generated by every $\lambda - 1$ of these generators has only the identity in common with the group generated by the remaining operator. For an abelian group whose order is a power of a prime number the number of the different operators in a possible set of independent generators is the same under both of the given definitions of a set of independent generators. The fact that the number of independent generators of a group is not always an invariant of the group becomes evident if we observe that when we generate the symmetric group of degree n by transpositions there will always be $n - 1$ independent generators. On the other hand, this symmetric group can also be generated by a cyclic substitution of degree $n - 1$ and a transposition involving the remaining letter. Complete proofs of these results are contained in a paper having the same heading, which has been offered for publication in the *Transactions of the American Mathematical Society.*